

An Explanation of Just Intonation

In order to understand just intonation it is important to understand equal temperament. The ratio for equal temperament is in relation to a power of 12- this is because human hearing/frequency is based on a logarithmic scale. Basically every note covers a slightly greater frequency range from the previous one so that the octaves are a perfect doubling (not adding) of frequency (110, 220, 440 as opposed to 110, 220, 330). This ratio keeps the semitones "equally" spaced easily allowing modulation between any of the 12 keys with instruments such as the piano and guitar that have "fixed tuning." This equal space between semi-tones is measured as 100 cents. Any number of cents sharp or flat is that percentage worth away from the equal spacing defined by this system of tuning.

While equal temperament is appropriate for many instruments, it is not ideal musically. Just intonation is the ideal system, however, to be practically used it requires flexible tuning to allow the use of all 12 keys (it requires 12 different pitches per note). It has not been commonly adopted because of these limitations, however, it is still ideal and can be easily used with many instruments, and so, arguably, should be used when possible.

So, what exactly is Just Intonation? It is a system of tuning that uses whole number ratios between intervals so that the harmonic series' of notes played line up causing the overtones of the notes to be in tune with one another. First, you must understand that every note played by an instrument contains other notes at differing volumes, referred to as harmonics. The specific volume differences of these harmonics are what create the timbre of different instruments (this is why instruments sound different even while playing the same note). So, every note played, in reality contains many notes, the loudest of which is generally the fundamental pitch (what we consider to be the note played). These harmonics are whole number multiples of the fundamental frequency. So, if a person plays an A3 at 220Hz there will be harmonics at A440Hz (*2), E660Hz (*3), A880Hz (*4), C#1100Hz (*5), etc. Because a justly tuned interval uses whole number ratios, its harmonics will line up with the whole number multiples of the root. For example, the justly tuned fifth at E330 would contain overtones of E660 (*2), B990 (*3), E1320 (*4), etc. The equal tempered fifth is "in tune" (i.e. equally spaced from all other notes) at 329.6276Hz. It would contain overtones of E659.26 (*2), B988.89 (*3), E1318.52 (*4), etc. These are obviously not too far off, but realize that the fifth is the closest to accurate of any interval in equal-tempered tuning. The root A contains an overtone of 660, whereas the 5th has an overtone of 659.26. These overtones are out of tune, yet being heard together at the same time. By playing "perfectly in tune" according to this system of tuning the overtones are always going to be out of tune! However, if you use just intonation the overtones are going to be in tune with each other because of the "simplified" mathematical ratio between intervals. The difficulty with Just Intonation is that the frequency of the note played depends on the key that it is played in. So, while A is at 440 Hz when it is the fundamental/root of a chord if

we change to an F major chord where A is the third it must be played at 436.5353Hz so that the interval is still a whole number ratio. This is a difference of -13.69 cents, which will remain true for all major thirds.

For instruments with flexible tuning this difficulty can be easily overcome by learning to change pitch with the changing function of a chord (third, fifth, seventh, etc). While tuning is ideally done by ear, it is possible to use a tuner by knowing the cent differences for each function of a chord. Voice is best suited for just intonation but many other instruments are flexible enough to use this system of tuning. These include, but are not limited to, the strings (violin, viola, cello, double bass), the brass (trumpet, trombone, euphonium, tuba), the woodwinds (flute, clarinet, saxophone, oboe, bassoon), and some percussion such as tympani. The strings attain this pitch flexibility by slightly altering the finger position. The wind instruments all can attain this flexibility through the player changing his/her embouchure. Many of the wind instruments also have other ways to easily change the pitch (trombones can slightly alter the slide position, flutes can roll in or out, etc). Below is a chart showing the ratios of equal temperament and just intonation and the relative cent difference for each interval.

| Note | Interval | Equal Ratio | Just Ratio | Cent Difference |
|------------|-------------|---------------------------------|------------|-----------------|
| A3 | Root | $2^{0/12}:1$ | 1:1 | 0 |
| A#3 | m2 | $2^{1/12}:1$ | 16:15 | +11.73 |
| B3 | M2 | $2^{2/12}:1$ | 9:8 | +3.91 |
| C4 | m3 | $2^{3/12}:1$ | 6:5 | +15.64 |
| C#4 | M3 | $2^{4/12}:1$ | 5:4 | -13.69 |
| D4 | P4 | $2^{5/12}:1$ | 4:3 | -1.96 |
| D#4 | TT | $2^{6/12}:1$ | 10:7 | +17.49 |
| E4 | P5 | $2^{7/12}:1$ | 3:2 | +1.96 |
| F4 | m6 | $2^{8/12}:1$ | 8:5 | +13.69 |
| F#4 | M6 | $2^{9/12}:1$ | 5:3 | -15.64 |
| G4 | d7 | $2^{10/12}:1$ | 7:4 | -31.17 |
| G#4 | M7 | $2^{11/12}:1$ | 15:8 | -11.73 |
| A4 | P8 | $2^{12/12}:1$ | 2:1 | 0 |